IAAO Annual Conference

Tampa, Florida

August 28-31, 2016

Estimating Depreciation from a Repeat Sales Model

Weiran Huang, PhD
Department of Finance
City of New York

August 29th, 2016

Basics of Depreciation

Depreciation : Decline in asset prices due to the aging of asset (Hulten and Wykoff 1981)

3 Categories:

1. Physical Deterioration
2. Functional Obsolescence
3. Economic Obsolescence

Methods of Estimating Depreciation

- Sales Comparison Method
- Capitalization of Income Method
- Overall Age-Life Method
- Engineering Breakdown Method
- Observed Condition Breakdown Method

Standard Repeat Sales Model

- First Sale $P_t = e^{\gamma_t} f\left(X_t; \beta_t\right)$
  - ✓ P_t: purchase price in period t
  - ✓ $f(X_t; \beta_t)$: unknown function of period-specific characteristics of the home (X) and their shadow price (β)
  - ✓ e^{γ_t}: the influence of period-specific market conditions that are common to all properties in the geographic market
- Second Sale $P_{t+\tau} = e^{\gamma_{t+\tau}} f(X_{t+\tau}; \beta_{t+\tau})$
- No physical change between these two sales

$$P_{t+\tau} = e^{\gamma_{t+\tau} - \gamma_t} P_t$$

$$\log \frac{P_{t+\tau}}{P_t} = \gamma_{t+\tau} - \gamma_t + \varepsilon_{t+\tau}$$

Standard Repeat Sales Model(continued)

- $\log \frac{P_{t+\tau,i}}{P_{t,i}} = \sum_{t=1}^{\tau_i} \gamma_t D_{t,i} + \varepsilon_{t,i}$ for observation i=1,2,…,n
- $D_{t,i}$ year dummies

  in period t, it equals -1 if it sells for the first time

  1 if sells for the second time

  0 if not sold

- Paird Sale House price Indices

  S&P/Case-Shiller Home Price Indexes

  Freddie Mac and OFHEO House Price indexes

Data

- Rolling sales for single – three family homes in five boroughs of New York City
- Arms-length transaction: removing sales between family members, foreclosure sales, estate sales, corporate sales, government sales, etc.
- 67,704 Paired sales during 2000 Q1-2016 Q2
- Removed ones with reported major renovations between paired sales

Data Summary

Variable	N	25th Pctl	50th Pctl	75th Pctl	Mean	Std Dev
Holding period	67,704	2.00	4.00	7.00	5.00	3.37
Age at purchase	67,704	48.00	75.00	86.00	65.25	30.92
Age at sale	67,704	53.00	79.00	92.00	70.24	30.77
Purchase Price($)	67,704	269,000	370,000	509,000	438,151	504393
Sale Price($)	67,704	367,000	479,000	635,000	578,227	761529

Age-related depreciation

- Price Change = Inflation + Net of Maintenance Depreciation

$$r_{price\ change} = r_i - \boxed{(r_{dep} - r_{maint})}$$

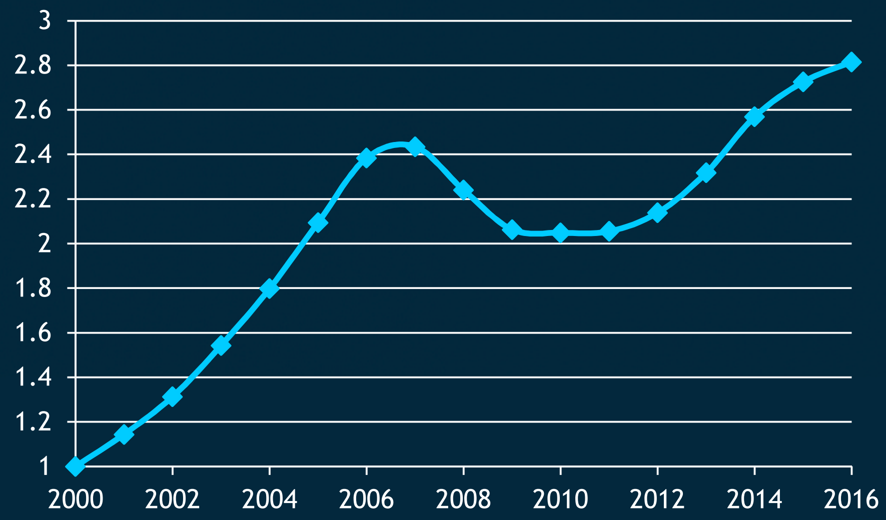
- Collinearity – nonlinear depreciation function
- Model A: $\log \tau$ (Lee, Ching and Kim 2005; Harding, Rosenthal and Sirmans 2007)

$$\log \frac{P_{t+\tau,i}}{P_{t,i}} = \boxed{\sum_{t=1}^{\tau_i} \gamma_t D_{t,i}} + \boxed{\alpha \log \tau_i} + \varepsilon_{t,i}$$

Price inflation

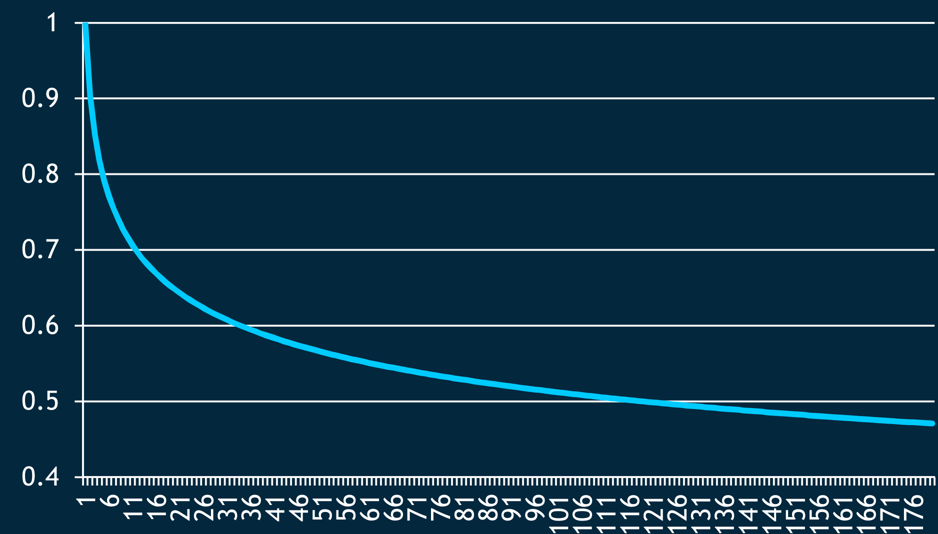
Depreciation

- α is the elasticity of housing price depreciation with respect to the change in age between sale dates



Price Inflation

Depreciation Curve



Model B: Depreciation with Age Groups

- Depreciation – continuously
- Maintenance - cyclical
- $\log \frac{P_{t+\tau,i}}{P_{t,i}} = \sum_{t=1}^{\tau_i} \gamma_t D_{t,i} + \alpha_j \left(D_{agp_j} * \log \tau_{j,i} \right) + \varepsilon_{t,i}$
- Different Age Groups
  - ✓ 0-10, 11-20, …, 110+
  - ✓ 0-10, 11-20, …, 111-120, 121-150, 150+
  - ✓ 0-5, 6-12, 13-20, 21-30, …, 110+
  - ✓ …
- $D_{agp_j} = 1$ if part of years $t + \tau$ belongs to this age group, otherwise $D_{agp_j} = 0$

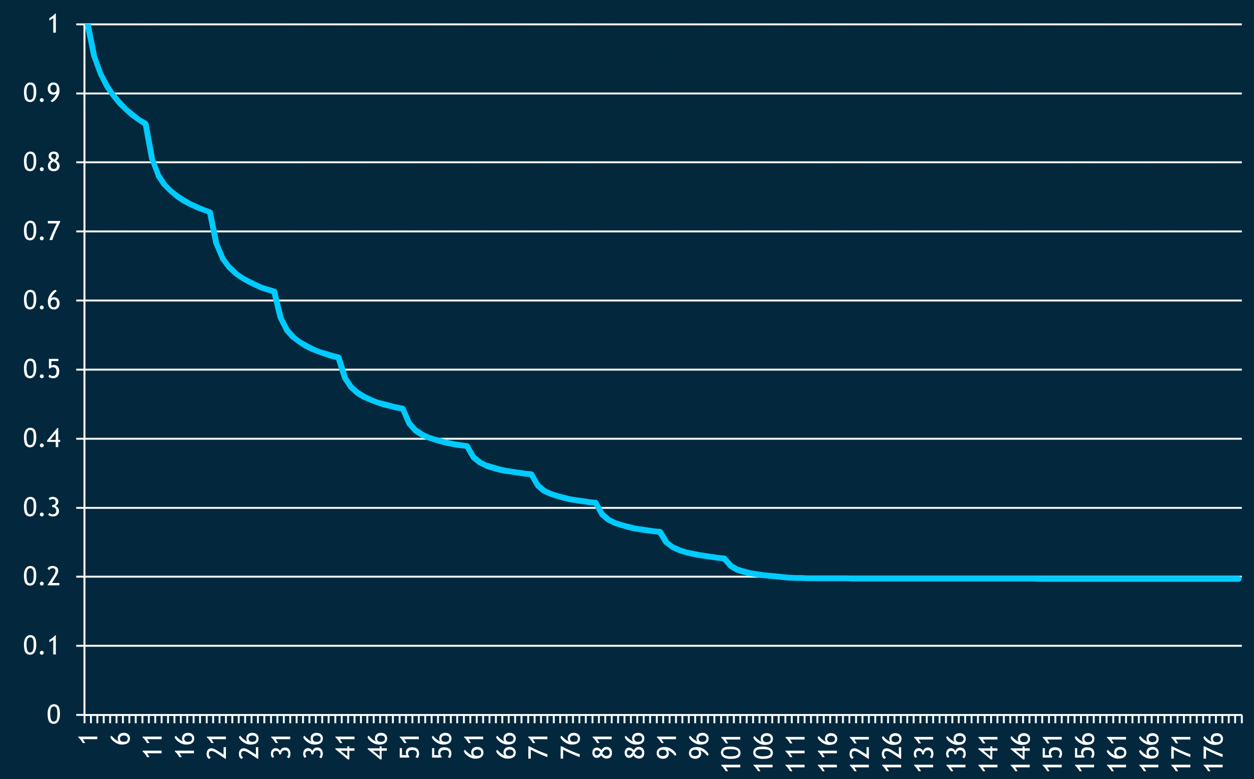
An Example

- $\log \frac{P_{t+\tau,i}}{P_{t,i}} = \sum_{t=1}^{\tau_i} \gamma_t D_{t,i} + \alpha_j \left(D_{agp_j} * \log \tau_{j,i} \right) + \varepsilon_{t,i}$
- A house built in 1980 was sold in 2003 and in 2014.
  - ✓ age=23 for the 1st sale, and age=34 for the 2nd sale
  - ✓ depreciation for 11 years belongs to two age groups
  - ✓ Depreciation function: $\alpha_2 \log 7 + \alpha_3 \log 4$

Housing Price Depreciation in Log-log Regression Models

	Model A			Model B		
Variable	Parameter Estimate	t-ratio	Pr>|t|	Parameter Estimate	t-ratio	Pr>|t|
τ(1-max)	-0.1451	-30.8	<.0001			
τ(1-10)				-0.0677	-25.94	<.0001
τ(11-20)				-0.0438	-16.61	<.0001
τ(21-30)				-0.0467	-15.07	<.0001
τ(31-40)				-0.0458	-18.58	<.0001
τ(41-50)				-0.042	-16.28	<.0001
τ(51-60)				-0.0354	-13.23	<.0001
τ(61-70)				-0.03	-12.22	<.0001
τ(71-80)				-0.0341	-14.78	<.0001
τ(81-90)				-0.0401	-16.45	<.0001
τ(91-100)				-0.0422	-14.67	<.0001
τ(101-110)				-0.0355	-10.43	<.0001
τ(110+)				-0.0017	-0.26	0.7913

Depreciation Curve with 10-year Group



Two-step Linear Depreciation

- Adjust sales price by inflation using Housing Price Inflation Index from the nonlinear model in the 1st step, then calculate depreciation in the 2nd step

- Model C: $\log \frac{P_{t+\tau,i}}{P_{t,i}} = \alpha \tau_i + \varepsilon_{t,i}$

- Model D: $\log \frac{P_{t+\tau,i}}{P_{t,i}} = \alpha_j \left(D_{agp_j} * \tau_{j,i} \right) + \varepsilon_{t,i}$

IAAO 2016

Housing Price Depreciation in Log-linear Regression Models

<table>
<tr><th></th><th colspan="3">Model C</th><th colspan="3">Model D</th></tr>
<tr><th></th><th>Parameter Estimate</th><th>t-ratio</th><th>Pr>|t|</th><th>Parameter Estimate</th><th>t-ratio</th><th>Pr>|t|</th></tr>
<tr><td>τ(1-max)</td><td>-0.0178</td><td>-45.97</td><td><.0001</td><td></td><td></td><td></td></tr>
<tr><td>τ(1-10)</td><td></td><td></td><td></td><td>-0.0298</td><td>-41.71</td><td><.0001</td></tr>
<tr><td>τ(11-20)</td><td></td><td></td><td></td><td>-0.0215</td><td>-36.09</td><td><.0001</td></tr>
<tr><td>τ(21-30)</td><td></td><td></td><td></td><td>-0.0222</td><td>-22.18</td><td><.0001</td></tr>
<tr><td>τ(31-40)</td><td></td><td></td><td></td><td>-0.0208</td><td>-19.47</td><td><.0001</td></tr>
<tr><td>τ(41-50)</td><td></td><td></td><td></td><td>-0.0181</td><td>-23.33</td><td><.0001</td></tr>
<tr><td>τ(51-60)</td><td></td><td></td><td></td><td>-0.0148</td><td>-22.86</td><td><.0001</td></tr>
<tr><td>τ(61-70)</td><td></td><td></td><td></td><td>-0.0108</td><td>-13.43</td><td><.0001</td></tr>
<tr><td>τ(71-80)</td><td></td><td></td><td></td><td>-0.0141</td><td>-20.15</td><td><.0001</td></tr>
<tr><td>τ(81-90)</td><td></td><td></td><td></td><td>-0.0163</td><td>-24.89</td><td><.0001</td></tr>
<tr><td>τ(91-100)</td><td></td><td></td><td></td><td>-0.0194</td><td>-18.51</td><td><.0001</td></tr>
<tr><td>τ(101-110)</td><td></td><td></td><td></td><td>-0.01</td><td>-5.99</td><td><.0001</td></tr>
<tr><td>τ(110+)</td><td></td><td></td><td></td><td>-0.0007</td><td>-0.17</td><td>0.8664</td></tr>
</table>

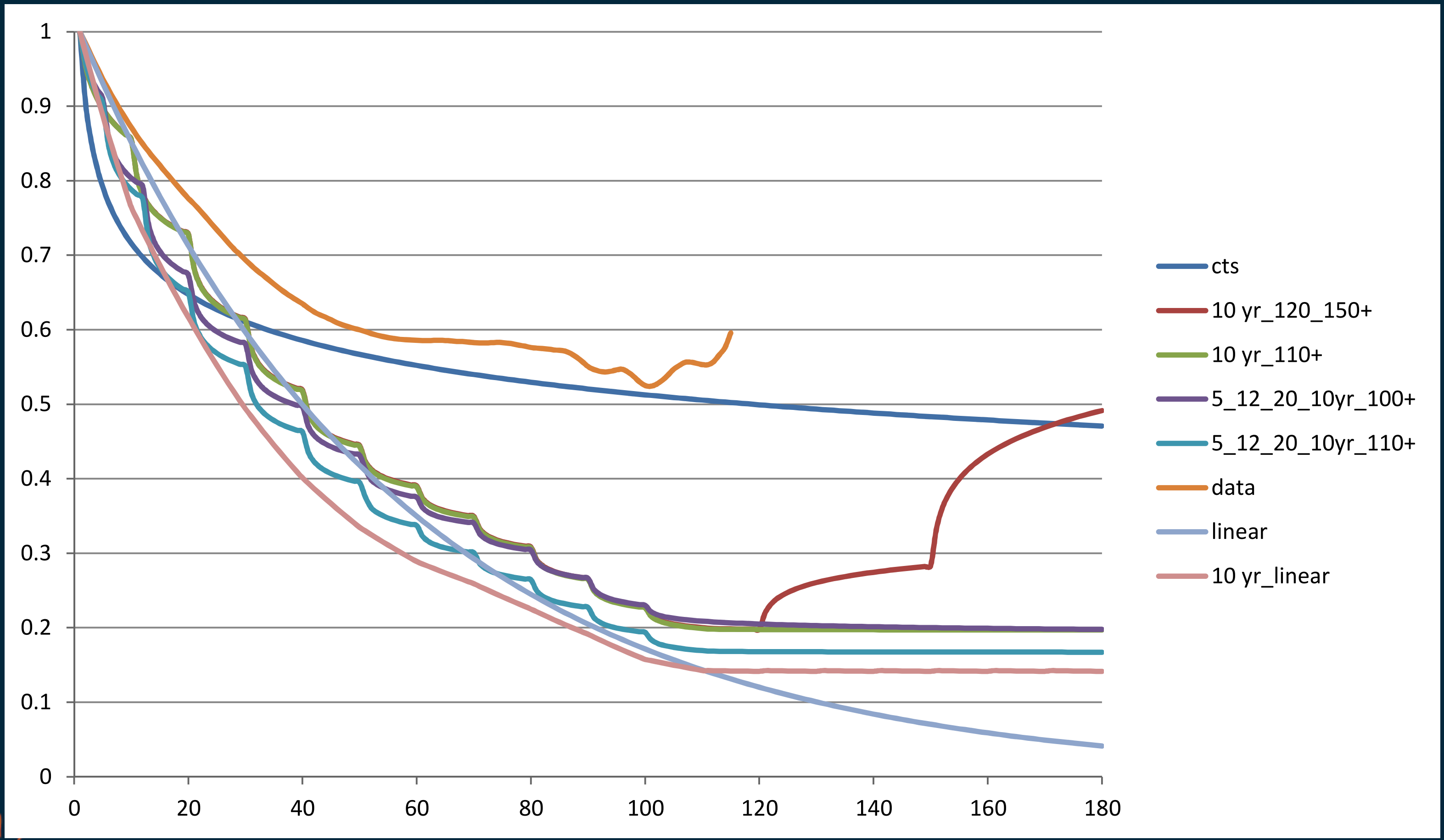
Median Depreciation Rate in Sample

- First adjust sales price by inflation, then measures the price changes as they age from the 1st sale to the 2nd sale

- $Depreciation\ rate = \frac{Sale\ 1 - Sale\ 2}{Holding\ Period\ in\ Years}$

Age	4	5	6	7	8	9	10
No. of Sales	3,735	3,466	3,128	2,799	2,485	2,204	1,897
Median	1.66%	1.60%	1.53%	1.48%	1.43%	1.36%	1.36%
Mean	1.11%	1.03%	1.05%	1.09%	1.16%	0.70%	0.78%

Depreciation Curves



Summary

- Repeat Sale Model give us a lot of options to model depreciation- use your own judgments
- Results agree with leading providers of building cost data
- We further use this depreciation schedule in our cost approach for single – three family homes in the borough of Brooklyn, model B achieves the best horizontal and vertical equity